# ASYMPTOTIC STABILITY METHOD FOR PID CONTROLLER TUNING IN A BACKHOE MACHINE


**Carlos Mastalli**
Mechatronic Research Group
Department of Process and System
Simon Bolivar University
Caracas, Venezuela
Email: carlosmastalli@usb.ve

**Dimitar Ralev**
**Novel Certad**
**Gerardo Fernandez-Lopez**
Mechatronic Research Group
Simon Bolivar University
Caracas, Venezuela



## ABSTRACT

This paper presents the modeling and study of dynamic behavior of a backhoe machine for tuning of PID controller. The tuning procedure of PID controller is performed, in detailed, for the case of a typical operation, digging a foundation and truck loading. This tuning procedure guarantees the local asymptotic stability in the sense of Lyapunov of origin of the closed-loop equation of PID controller. Besides the tuning procedure requires the knowledge of certain properties of dynamic model, which are dependent on the desired trajectory. Finally it is demonstrated that this tuning procedure proves to be effective, and also robust in the execution of other tasks performed by the backhoe machine.


## 1 Introduction

In the field robotics, there is a well defined tendency for development of technologies in the fields of motion planning, perception, navigation and control that allow the complete autonomy of machines. Such techniques are being improved and applied in robots with dynamics, complex and non-parametrized workspaces. In this sense, the motion planners require the development of control systems that guarantee, in an efficient and robust way, the autonomous execution of task in backhoe machines. Because it is vital for the security of operations in heavy machineries.

On the other hand, the development of control systems in backhoe machines presents important challenges due to the non-linearities of dynamic model and the external forces product of the interaction between the soil and tool. In [1] a robust impedance control for manipulator robots is developed, the force is regulated by controlling the position and its relationship (impedance) with the force. This kind of control, impedance control, was implemented for the automation of excavation tasks in backhoe machines by [2]. In this regard, in this work is performed a dynamic and kinematic analysis of the backhoe machine for PID controllers tuning. The tuning procedure used is determined in [3] through the stability analysis in the sense of Lyapunov of PID controller. Moreover, such procedure is based on the knowledge of dynamic model of the machine, the tuning procedure is detailed in this paper.

Finally, Section 2 describes the mathematical dynamic model of backhoe arm, where it runs a set of tests to validate this model. In Section 3 the implementation of backhoe arm control system is shown. Besides, a detailed procedure of tuning for the PID controllers is described which is based in asymptotic stability method in the sense of Lyapunov. Finally, in Section 4 the discussion of the obtained results and future works is presented.

## 2 Dynamic Model of Backhoe Arm

For the development of dynamic model of backhoe arm a kinematic model of machine is required. In this sense, the first step consists in selecting the references frames, using the Denavit-Hartenberg (DH) convention [4], for the determination of the DH parameters. In the Fig. 1[1] the assignment of the references frames is shown, according to the DH convention, for the construction of kinematic and dynamic model of backhoe arm.

[1]Note that $m_i$, $I_i$ and $cm_i$ represents the mass, mass moment of inertia and the position of center of mass of link $i$, respectively.

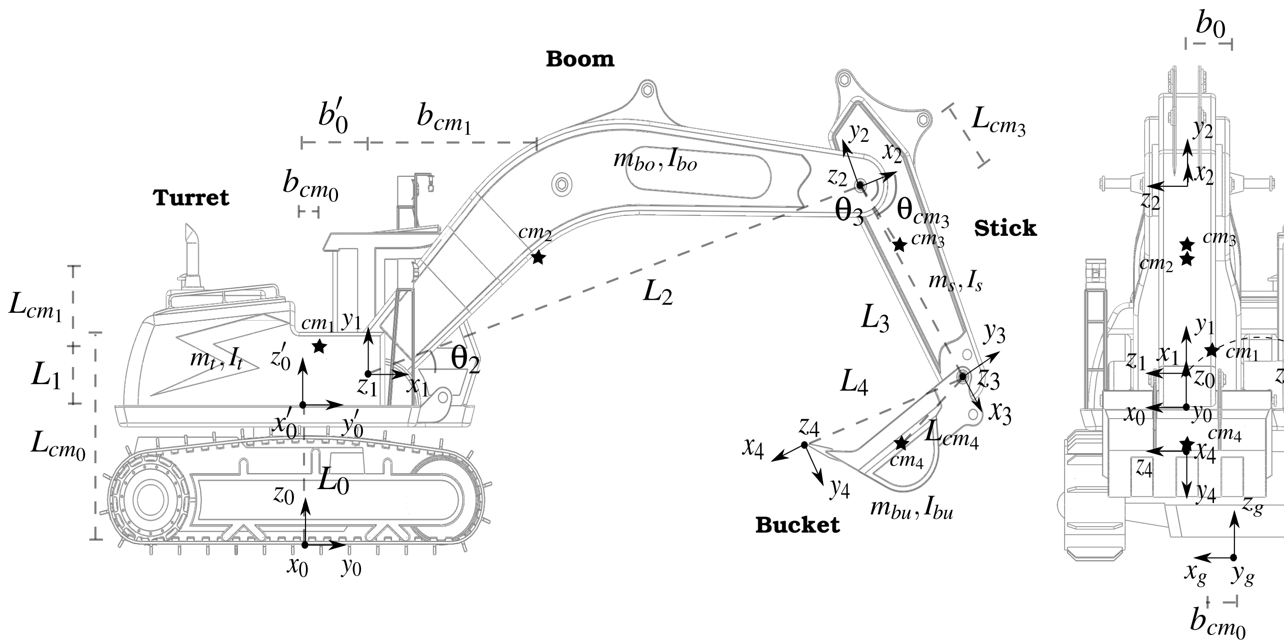


Figure 1: Assignment of the references frames of backhoe, according to Denavit-Hartenberg parameters.

The model developed in this paper assume that the hydraulic actuators act as infinitely powerful force sources, in the same way that was presented in [5] [6].

Therefore, it develops a conservative dynamic model (1), which does not consider non-conservative forces such as: viscous and static friction forces, then it follows that:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau \tag{1}$$

## 2.1 Validation of Backhoe arm Dynamic Model

In this paper, a set of test are using to validate the dynamic model of backhoe arm. These test units are made through the implementation of direct dynamic model of machine arm, i.e.

$$\ddot{\mathbf{q}} = \mathbf{M}(\mathbf{q})^{-1}(\tau - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q})) \tag{2}$$

where the non-linear ordinary differential equation is solved in a numerically form by using trapezoidal integration of (2).

The following provides 3 case studies designed to analyze the dynamic response of backhoe arm according to a set of conditions, initial position and input torque, parameterized. These conditions are selected to compare the natural behavior of the backhoe arm with the integration of dynamic model (2).

**2.1.1 Case study 1 - Falling movement** In this case study, the backhoe arm is subject to small input torque[2], $\tau = \left[0\ 114.3\ -1.8\ -3.2\right]^T \cdot 10^3$ Nm, that is not sufficient to maintain the position of backhoe arm; besides the initial articular position is $\mathbf{q} = \left[0\ 0\ -10\ 0\right]^t$ degrees. Accordingly, in this condition a falling movement of arm due to the gravitational torque is expected. Therefore, Fig. 2 depicts the backhoe-arm movement with the conditions mentioned, where the green lines represents the initial state of arm, and the red ones and red points represents the state of links and joints, respectively.

In Fig. 2 the links movements sequence (red lines) exposes a falling movement of backhoe arm. In this falling movement is observed that the dynamic of the first link, the longest link, is dominant because the gravitational torque in this joint is greater, i.e. $\tau_0 = \left[0\ 152.4\ -2.4\ -4.27\right]^T \cdot 10^3$ Nm. Besides it can be seen, due to Coriolis forces, a chain movement between each one of the links. Note also that the physical constraints of each joint in the dynamic model implemented is not consider. Finally, it is shown that the falling movement of backhoe arm occurs in the *xy* plane, which is the expected movement.

**2.1.2 Case study 2 - Falling movement with torque in the turret** This case study is similar to the previous one but it adds significant input torque in the first or turret joint. Thus, it is expected that the movement of backhoe arm be similar to Case 1 but with a rotation of turret. Besides the starting conditions are the same as Case 1, i.e. $\mathbf{q} = \left[0\ 0\ -10\ 0\right]^t$, but with input torque $\tau = \left[500\ 114.3\ -1.8\ -3.2\right]^T \cdot 10^3$ Nm. In this sense, in Fig. 3 is observed the behavior of backhoe arm, i.e. it maintains the shape of the falling movement but associating a rotational component of movement in the direction of the turret input torque.

[2] Compared with the gravitational torque in the initial position, i.e. $\tau_0 = \left[0\ 152.4\ -2.4\ -4.27\right]^T \cdot 10^3$ Nm.

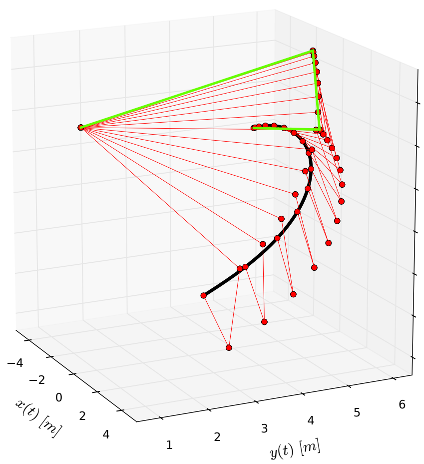


Figure 2: Case study 1 - Falling movement sequence of backhoe arm.

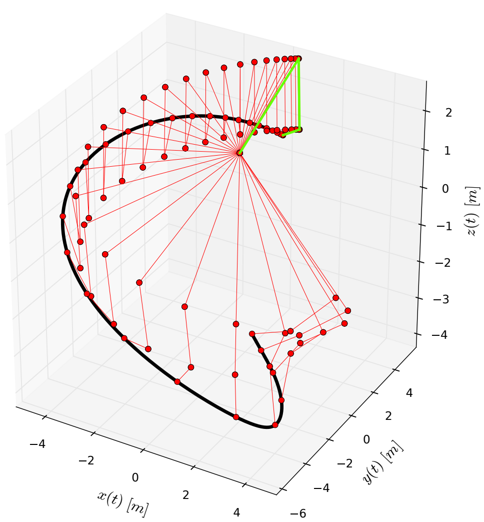


Figure 3: Case study 2 - Falling movement sequence with torque in the turret of backhoe arm.

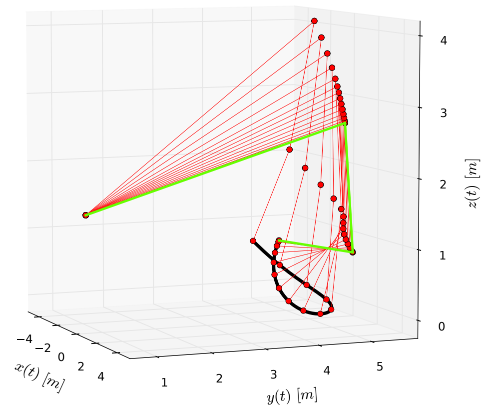


Figure 4: Case study 3 - Falling movement sequence with torque in the boom of backhoe arm.

#### 2.1.3 Case study 3 - Falling movement with torque in the boom

In this case an input torque is supplied, significant in the boom joint, which correspondences to $\tau = \left[0\ 155.0\ -1.8\ -3.2\right]^T \cdot 10^3$ Nm that is greater than gravitational torque in the initial position. Then, it is expected that boom link move continuously upwards until the equilibrium point.

In Fig. 4 is appreciated the movement sequence for this case study, where the boom link move up as it was expected. Besides, it is shown that bucket link move downward when boom link move upward. This results are coherent with the real behavior of backhoe arm.

Finally it is demonstrated, in each case study, the validity of the analytical dynamic model developed for backhoe arm, that it will be used for the tuning procedure of PID controllers.

## 3 Backhoe-arm Control System

For the purposes of motion planning, it is convenient to encode the backhoe tasks in the workspace of the machine. In [7] an imitation learning approach is developed as motion planners, and this motion planning algorithm encode the backhoe tasks in the workspace of machine. Therefore, it is required the implementation of operational space control that allows the appropriate execution of planned movement. In [8] a theoretical and empirical comparison of operational space control for the complex case of redundant manipulators is done. Such implementation is based in the use of Proportional-Integral-Derivative (PID) controllers in each rotational joint[3] of the backhoe arm. In Fig. 5 can be seen that the control system implemented has two main components. The first component consists in the implementation of the inverse kinematic algorithm [9] [10] that allows mapping the generate movement, in the codification space of imitation learning approach $\left[x_e\ y_e\ z_e\ \psi_e\right]^T$, to joint space. The second component consists in the implementation of control system that is based on PID controllers.

In Eq. (3) is described the digital control law that governs the movement of backhoe arm. In this PID controller, the computation of integral and derivative action depends of sample time of error signal $\Delta t$. The implementation of control system (see Fig. 5) is based in a hard real-time mechanism[4], where the com-

[3] The dynamic of hydraulic actuators have not been consider in this work.

[4] In contrast to soft real-time systems, hard real-time system must comply

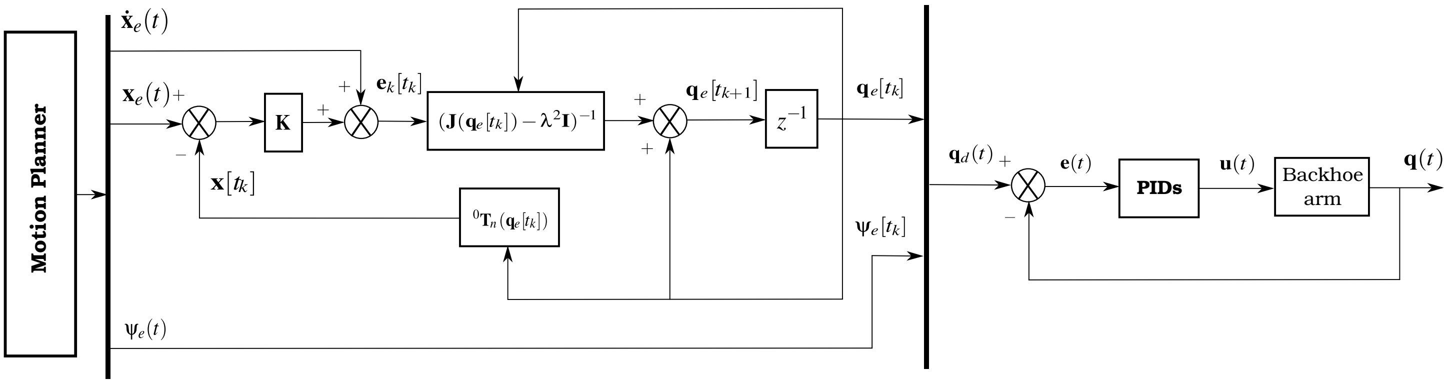


Figure 5: Backhoe-arm Control System.

pute loop is performed at 1000 Hz and the joint sensing at 100 Hz, i.e. it has $\Delta t = 1$ mseg.

$$u(t_k) = K_p \left[ e(t_k) + \frac{1}{T_i} \sum_{i=1}^{k} e(t_i)\Delta t + T_d \left( \frac{e(t_k) - e(t_{k-1})}{\Delta t} \right) \right] \quad (3)$$

where $K_p$ is proportional gain, $T_i$ is integral time parameter, $T_d$ is derivative time parameter and $\Delta t$ is the sample time of the error signal.

### 3.1 PID controller tuning

In Eq. (1) can be seen that backhoe-arm dynamic model is non-linear, besides the PID control law (3) is a lineal strategy. Therefore, it will require a convenient tuning of the PID controller parameters ($K_p$, $T_i$ and $T_d$) that guarantee the asymptotic stability of the origin, because the linear control action does not cancel the non-linear components of backhoe-arm dynamic.

In [3] the asymptotic stability of PID controller is analyzed, and it is shown that the stability of origin is in local way, i.e. for boundary joint velocities $\|\dot{\mathbf{q}}\|$, where depends of the selection of PID controller gains.

$$\|\dot{\mathbf{q}}\| < \frac{1}{k_{C1}} \left[ \frac{\lambda_{\min}\{\mathbf{K}_d\}[\lambda_{\min}\{\mathbf{K}_p\} - k_g]}{\lambda_{\max}\{\mathbf{K}_i\}} - \lambda_{\max}\{\mathbf{M}\} \right] \quad (4)$$

and this result is achieved when the position $\tilde{\mathbf{q}}(0)$ and velocity $\dot{\tilde{\mathbf{q}}}(0)$ errors are sufficiently small [11].

In addition, according to the first method of Lyapunov stability is necessary to comply with the conditions (5) to guarantee the local asymptotic stability of the origin.

$$\begin{aligned} \lambda_{\max}\{\mathbf{K}_i\} &\geq \lambda_{\min}\{\mathbf{K}_i\} > 0 \\ \lambda_{\max}\{\mathbf{K}_p\} &\geq \lambda_{\min}\{\mathbf{K}_p\} > k_g \\ \lambda_{\max}\{\mathbf{K}_d\} &\geq \lambda_{\min}\{\mathbf{K}_d\} > \frac{\lambda_{\max}\{\mathbf{K}_i\}}{\lambda_{\min}\{\mathbf{K}_p\} - k_g} \frac{\lambda^2_{\max}\{\mathbf{M}\}}{\lambda_{\min}\{\mathbf{M}\}} \end{aligned} \quad (5)$$

where

$$k_g \geq n \left[ \max_{i,j,q} \left| \frac{\partial g_i(\mathbf{q})}{\partial q_j} \right| \right] \quad (6)$$

and

$$k_{C1} \geq n^2 \left[ \max_{i,j,k,q} |C_{kij}(\mathbf{q})| \right] \quad (7)$$

are intrinsic parameters of dynamic behavior of backhoe arm, where $n$ represents the number of degrees of freedom (dof).

As shown in (5), the tuning procedure requires the knowledge of inertial matrix $\mathbf{M}(\mathbf{q})$ and gravitational vector $\mathbf{g}(\mathbf{q})$ of backhoe arm, in order to determine $\lambda_{\min}\{\mathbf{M}\}$, $\lambda_{\max}\{\mathbf{M}\}$ and $k_g$, respectively. Besides, it is noted that the attraction to equilibrium will be increment when the inequalities (5) are fulfilled more strongly[5].

On the other hand, a tuning procedure of PID controller using a Lyapunov stability implies that there exist an invariant set for the closed-loop dynamic, i.e

$$\Omega = \{\mathbf{q} \in \mathbb{R}^n, \dot{\mathbf{q}} \in \mathbb{R}^n : V(\mathbf{q}, \dot{\mathbf{q}}) \leq \alpha\} \subseteq \mathcal{Q} \quad (8)$$

and hence for any initial state $\mathbf{q_0} \in \Omega$ and $\dot{\mathbf{q}}_\mathbf{0} \in \Omega$, the system fulfills the constraints and remains inside $\Omega$, where $\mathbf{q} \in \mathcal{Q}$ is constrained by the polytope $\mathcal{Q}$.

within established deadlines; otherwise a fault will occur.

[5]This attribute is known as semi-global attractiveness.

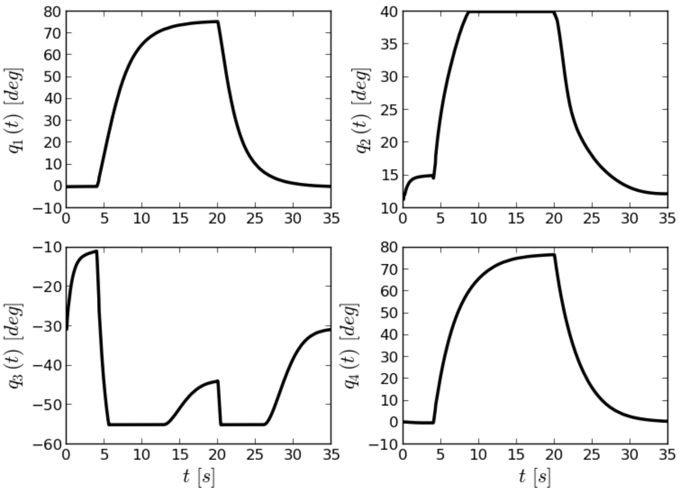


Figure 6: Joint positions of a typical movement of backhoe arm.

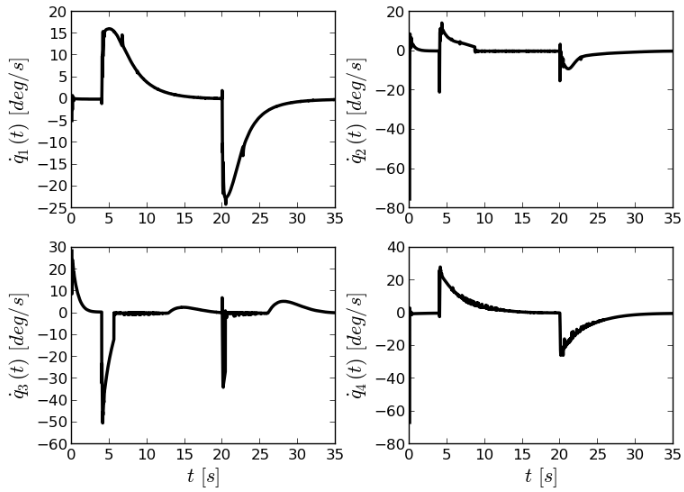


Figure 7: Joint velocities of a typical movement of backhoe arm.

**Movement for PID controller tuning** A typical movement of backhoe machine will be used for PID controller tuning. For purpose of tuning, the selected movement represents a truck loading operation, where this movement has three main phase; the first one corresponds to a standby movement, the second ones corresponds to a truck loading operation and the last one corresponds to a return movement.

In Fig. 6 and 7 the corresponding joint movements for the tuning procedure are shown. These figures shows clearly three phase of movement, $0-5$ *sec*., $5-20$ *sec*. and $20-35$ *sec*., which corresponds to the aforementioned movement.

According to Fig. 7 the norm of joint velocity signal $\|\dot{\mathbf{q}}\|$ is determined. The maximum value of $\|\dot{\mathbf{q}}\|$ is important to design a controller with asymptotic stability in this locality. Besides in Fig. 8 can be seen that the controller must be asymptotically stable in the origin for velocities up to $\|\dot{\mathbf{q}}\| = 59.53\ deg/s$, where this boundary velocity is good enough for any kind of operations.

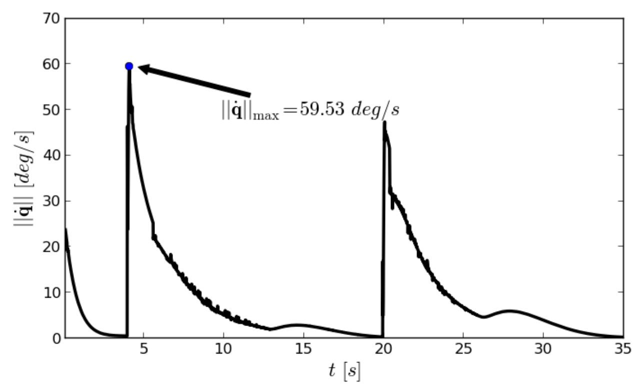


Figure 8: Norm of joint velocity for a typical movement of backhoe arm.

**Dynamic model parameters of backhoe arm** As is seen in Eq. (4) and (5), it is required to compute some parameters that depends of movement of backhoe arm. Therefore, the parameters that must be calculated are $k_g$, $k_{C1}$, $\lambda_{\min}\{\mathbf{M}(\mathbf{q})\}$ and $\lambda_{\max}\{\mathbf{M}(\mathbf{q})\}$, where (6) and (7) are the equations to calculate $k_g$ and $k_{C1}$, respectively.

According to Eq. (6), the results of computing of maximum component of partial derivative of gravitational vector with respect to $q$ $\max_{i,j,q}\left|\frac{\partial g_i(\mathbf{q})}{\partial q_j}\right|$, for the movement described by Fig. 6 and 7, are shown in Fig. 9. Thus, the maximum point occurs in $t = 19.91\ s$ with a value of $6017\ kg\,m^2/s^2$.

On the other hand, the computing of component $\max_{i,j,k,q}\left|C_{k_{ij}}(\mathbf{q})\right|$ of $k_{C1}$ is shown in Fig. 10. Besides both values, the maximum absolute value of derivative of gravitational vector with respect to joint position $\mathbf{q}$ and Coriolis matrix, are localed around $t = 20\ s$; indeed the higher values occurs for the movement phase of truck loading.

Moreover, a pseudo-proportional increase and decrease between $\left|\frac{\partial g_i(\mathbf{q})}{\partial q_j}\right|$ and $\left|C_{k_{ij}}(\mathbf{q})\right|$ is shown; characteristic that could be mapping in any tasks performed by a backhoe machine. Therefore, this pattern facilitates the determination of tuning criterion of PID controllers, i.e. $k_g$ and $k_{C1}$ are calculated based on maximum values found around $t = 20\ s$.

Then it is assigned $k_g = 24068\ kg\,m^2/s^2$ and $k_{c1} = 113779.2\ kg\,m^2$ because $n = 4$ (number of d.o.f), and $\max_{i,j,q}\left|\frac{\partial g_i(\mathbf{q})}{\partial q_j(\mathbf{q})}\right| = 6017\ kg\,m^2/s^2$ and $\max_{i,j,k}\left|C_{k_{ij}}(\mathbf{q})\right| = 7111.2\ kg\,m^2$, respectively.

In connection with this maximum and minimum values of

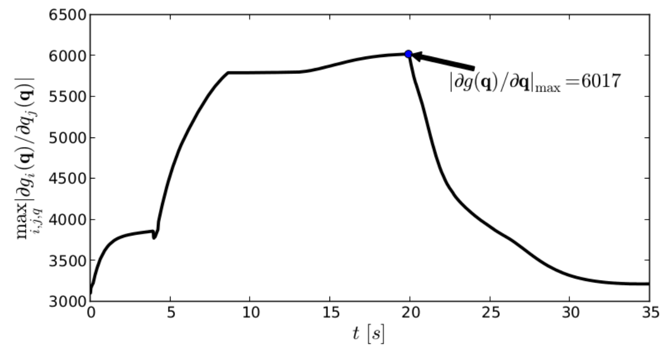


Figure 9: $\max_{i,j,q} \left| \frac{\partial g_i(\mathbf{q})}{\partial q_j} \right|$ of a typical movement of backhoe arm.

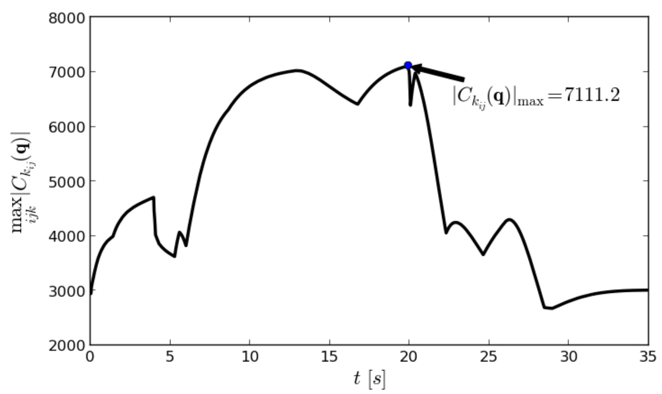


Figure 10: $\max_{i,j,k} \left| C_{k_{ij}}(\mathbf{q}) \right|$ of a typical movement of backhoe arm.

inertial matrix, in Fig. 11 the computation of these values for movement concerned is shown. In the same way that Fig. 9 and 10, in Fig. 11 can be observed a pseudo-proportional pattern between both signals, where their maximum values are in $t = 20$ *sec*, and with a local minimum $\lambda_{\min}\{\mathbf{M}(\mathbf{q})\}$ close in magnitude to the global minimum. Note that relationship $\frac{\lambda^2_{\max}\{\mathbf{M}(\mathbf{q})\}}{\lambda_{\min}\{\mathbf{M}(\mathbf{q})\}}$ is the most important component for the tuning procedure because this relationship defines the maximum value of conditions (5).

In Fig. 12 can be seen that global maximum occurs in $t = 8.37$ *sec*, and not around $t = 20$ *sec*, while there is a local maximum in $t = 20$ *sec* with value $1.6368 \cdot 10^5\ kg^2\, m^4$. However, it is decided to use an extreme values for PID tuning.

**Tuning procedure** The tuning procedure of gains $\mathbf{K}_p$, $\mathbf{K}_d$ and $\mathbf{K}_i$ is an iterative procedure based on conditions (4) and (5), where such equations only restrict the space of possible gains. Therefore, it was effectuated about 300 simulations in Gazebo

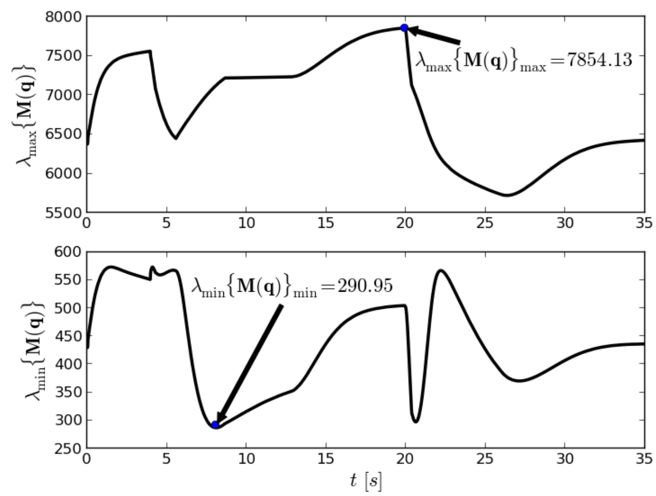


Figure 11: $\lambda_{\max}\{\mathbf{M}(\mathbf{q})\}$ and $\lambda_{\min}\{\mathbf{M}(\mathbf{q})\}$ of a typical movement of backhoe arm.

[12] to determine an appropriate tuning[6], which is based on conditions (5)

$$\mathbf{K}_p = \begin{bmatrix} 802500 & 0 & 0 & 0 \\ 0 & 782500 & 0 & 0 \\ 0 & 0 & 582500 & 0 \\ 0 & 0 & 0 & 372500 \end{bmatrix} [Nm/rad]$$

$$\mathbf{K}_d = \begin{bmatrix} 1295000 & 0 & 0 & 0 \\ 0 & 845000 & 0 & 0 \\ 0 & 0 & 685000 & 0 \\ 0 & 0 & 0 & 365000 \end{bmatrix} [Nm/rad/s]$$

$$\mathbf{K}_i = \begin{bmatrix} 54000 & 0 & 0 & 0 \\ 0 & 555000 & 0 & 0 \\ 0 & 0 & 455000 & 0 \\ 0 & 0 & 0 & 55000 \end{bmatrix} [Nm/rad\, s]$$

where the results of PID tuning for backhoe arm are shown in Table 1.

In the selection of parameters is ensured the asymptotic stability of selected movement (see Fig. 6), i.e. the local asymptotic stability is achieved for the boundary of $\|\dot{\mathbf{q}}\| < 1149.97\ deg/s$ because it is satisfied that:

$$\begin{aligned} \lambda_{\max}\{\mathbf{K}_i\} \geq \lambda_{\min}\{\mathbf{K}_i\} &> 0 \quad [Nm/rad\, s] \\ \lambda_{\max}\{\mathbf{K}_p\} \geq \lambda_{\min}\{\mathbf{K}_p\} &> 24068 \quad [Nm/rad] \\ \lambda_{\max}\{\mathbf{K}_d\} \geq \lambda_{\min}\{\mathbf{K}_d\} &> 280007 \quad [Nm/rad/s] \end{aligned} \tag{9}$$

[6] Note that PID values are higher because is not modeled the dynamic of hydraulic actuators..

| | $\mathbf{K}_p$ | $\mathbf{K}_d$ | $\mathbf{K}_i$ |
|---|---|---|---|
| $q_1$ | 802500 | 129500 | 54000 |
| $q_2$ | 782500 | 845000 | 555000 |
| $q_3$ | 582500 | 685000 | 455000 |
| $q_4$ | 372500 | 365000 | 55500 |

Table 1: PID values for backhoe arm.

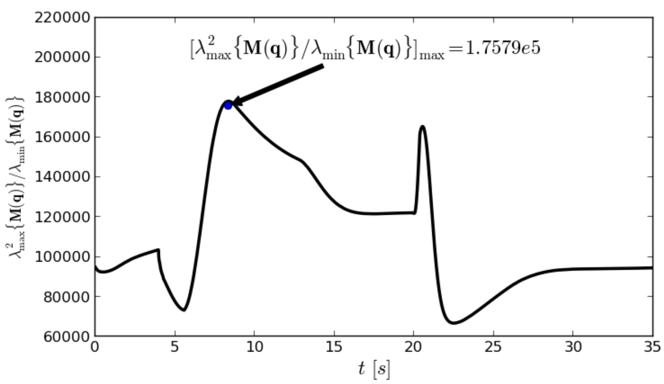


Figure 12: $\frac{\lambda^2_{\max}\{\mathbf{M}(\mathbf{q})\}}{\lambda_{\min}\{\mathbf{M}(\mathbf{q})\}}$ of a typical movement of backhoe arm.

Moreover, this tuning has been tested for other movements in simulation which has been planned through the implementation of the imitation learning approach [7]. The PID controller tuning has been demonstrated successful results for any planned operation in the backhoe machine.

In Fig. 13 the results of implementation and tuning of PID controllers are presented. In this figure can be seen that is achieved asymptotic stability in the origin for the response of control system. In the sections of $q_3(t)$ does not occur appropriate tracking because the physical constraint of stick link.

Besides, in Fig. 14 can observe, with detail, that the errors are bounded between $-2 < e(t) < 2$ degrees with exception of $e_3(t)$ and a section of $e_2(t)$. As mentioned above, this problem is not associated with PID controller tuning, but rather physical restrictions of backhoe arm.

Finally, in Fig. 15 the control signal of each one of the joints is shown; where it can see that the higher control effort happens in the stick link, i.e. in $q_2$ because of the physical restrictions of this joint. The higher control effort is associated with the higher gravitational torque, which occurs in the stick link.

## 4 Conclusion

The cartesian control of automated backhoe machines is a vary challenging problem, specially due to severe nonlinearities.

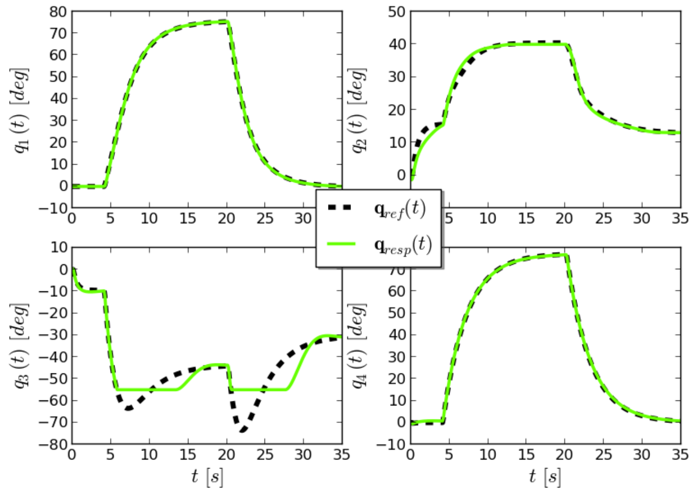


Figure 13: Response of PID control system for a typical movement.

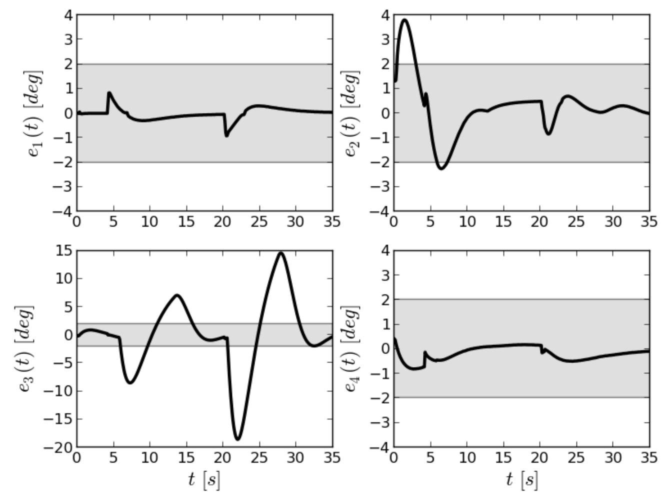


Figure 14: Error of PID control system for a typical movement.

The nonlinearities tend to increase with the size of the excavator, and also, with the speed of end-effector.

In this sense, this paper develops and analyzes a PID controller tuning procedure for the control systems of backhoe arm, as a simple methodology for tuning a PID controller. The tuning procedure is based in the desired movement of backhoe arm. The selected movement represents a typical operation of backhoe machines, i.e. truck loading operation. Numeric simulations demonstrate the effectiveness and security of the tuning procedure for the execution of operations in backhoe machines, which suggest the technical possibility of achieving autonomous

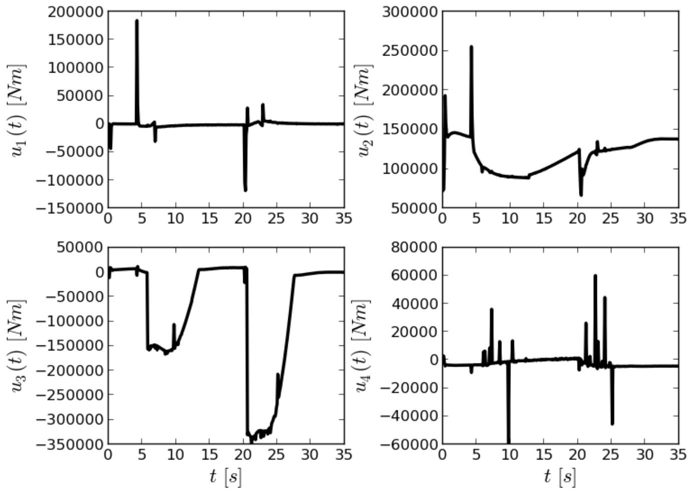


Figure 15: Control signal of PID control system for a typical movement.

robotic excavation in moving toward construction automation. In addition, it is demonstrated that relationship $\frac{\lambda^2_{\max}\{\mathbf{M}(\mathbf{q})\}}{\lambda_{\min}\{\mathbf{M}(\mathbf{q})\}}$ is the most influential component in the tuning procedure, because it is a measure of kinematic energy of the machine. Besides, this method guarantees robustness for $\mathbf{q} \in \mathcal{Q}$ because it is demonstrated that Lyapunov stability ensures an invariant set $\Omega$.

Finally, futures works will address the analysis of stability in the sense of Lyapunov for several external forces in the bucket, and also the modeling of these interaction forces between the soil and tool.

## REFERENCES


[1] Lu, Z., and Goldenberg, A., 1995. "Robust impendace control and force regulation: theory and experiments". *International Journal Robotic Research,* **14**(3), pp. 225–254.

[2] Shi, X., Lever, P., and Wang, F., 1996. "Experimental robotic excavation in construction". In Proceedings IEEE International Conference of Robotics and Automation, Vol. 1, pp. 957–962.

[3] Kelly, R., 1995. "A tunning procedure for stable pid control of robot manipulators". In Robotica, Vol. 13, pp. 141–148.

[4] Denavit, J., and Hartenberg, R., 1955. "A kinematic notation for lower-pair mechanisms based on matrices". *Transactions of ASME of Journal of Applied Mechanics,* **23**, p. 215–221.

[5] Vaha, P., and Skibniewski, M., 1993. "Dynamic model of an excavator". *Journal of Aerospace Engineering,* **6**, pp. 148–158.

[6] Koivo, A., 1994. "Kinematics of excavator (backhoes) for transferring surface material". *Journal of Aerospace Engineering,* **7**, pp. 17–31.

[7] Mastalli, C., Cappelletto, J., Acuña, R., Terrones, A., and Fernández-López, G., 2012. "An imitation learning approach for truck loading operations in backhoe machines". In 15th International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines.

[8] Nakanishi, J., Cory, R., Jan Peters, M., and Schaal, S., 2008. "Operational space control: A theorical and empirical comparison". *The International Journal of Robotic Research,* **27**(6), pp. 737–757.

[9] Buss, S. R., 2009. Introduction to inverse kinematics with jacobian transpose, pseudoinverse and damped least squares methods. Tech. rep., University of California, San Diego.

[10] Jovanovic, V. T., and Sendzimir, T., 1996. On the metrics and coordinate system induced sensitivity in computational kinematics. Tech. rep., Department of Mechanical Engineering. University of Connecticut.

[11] Arimoto, S., and Miyasaki, F., 1984. "Stability and robustness of pid feedback control for robot manipulator of sensory capability". In Robotics Research: First International Symposium, pp. 783–799.

[12] Koenig, N., and Howard, A., 2004. "Design and use paradigms for gazebo, an open-source multi-robot simulator". In IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 2149–2154.